Invited Lecture

Digital Technologies, Cultures and Mathematics Education

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ABSTRACT I focus here on several aspects of cultures related to digital technologies and mathematics education. One first aspect is that any integration of digital technologies for mathematical (or other) teaching and learning creates and transforms the classroom culture. On the other hand, in order for learning to be meaningful through the use of digital technologies, these may need to be embedded in a certain "culture" that empowers students to engage pro-actively with those technologies. I present different types of teaching and classroom "cultures" that have been found when using digital technologies, how these impact mathematical learning, as well as different conditions and teacher-training opportunities for the use of digital technologies found in different countries, illustrating all of these with examples from my own experience and from literature. I discuss how the different conditions and access opportunities in different regions and cultures create digital gaps. Finally, I discuss what could be done to support teachers to create meaningful contexts and classroom cultures when integrating digital technologies within established school systems (but at the same time transforming these), so that these can empower learners (e.g., to "do mathematics") and promote the construction of knowledge.

Keywords: Digital technologies; Classroom cultures; Mathematics education; Constructionism.

1. Introduction

I discuss here ideas and issues related to how different cultural aspects affect the integration of digital technologies in mathematics education.

Digital technologies permeate our lives, but I ask: How are they used for (mathematical) teaching and learning? Are they integrated in ways that promote significant learning and enhanced mathematical practices? I argue that they generally do not, except in exceptional cases. My aim here is to reflect, on the one hand, on how existing cultures affect how technologies are integrated in mathematical classrooms and for mathematical learning. On the other hand, when digital technologies are available and/or integrated into school practices, I reflect also on what kind of cultures are, or can be, created for students and teachers — in educational institutions and classrooms — to promote mathematical learning.

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Firstly, I believe that existing cultures affect how digital technologies are integrated for mathematics education and in its teaching and learning; secondly, that when digital technologies are integrated they create new cultures; and thirdly, that new cultures need to be created in order to have a significant and effective mathematics education impact.

That is, while it is important to create different school cultures for integrating digital technologies, at the same time, and conversely, digital technologies create new cultures in the classroom, as is illustrated in Fig. 1. This is part of what I will reflect upon.

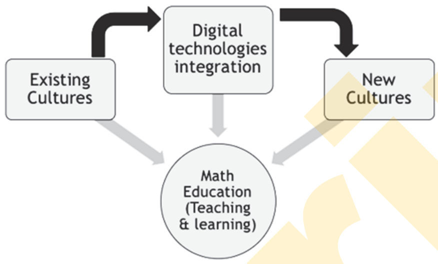

Fig. 1. Cultures and digital technologies integration (a two-fold process); and their effect on mathematics education

But I will also reflect (i) on the constraints and what I call the *inertia of classroom cultures*; (ii) on the barriers to meaningful and transformative technology integration in mathematics classrooms, which include digital gaps and issues of access and equity; as well as (iii) on teachers' cultures. In relation to the latter, I believe that teachers are key players for integrating digital technologies in the classroom and that there needs to be more teacher involvement in both the design of technological implementations as well as of related resources.

2. Towards A Significant Integration of Digital Technologies for Mathematical Learning: Revisiting the Constructionist Paradigm

My views related to the integration of digital technologies for mathematics teaching, learning and thinking, are informed in great part by the constructionist school of thought.

2.1. The core ideas of constructionism

Constructionism is a term coined by Seymour Papert to describe a paradigm of learning based on the tenet that building knowledge structures in the mind "happens especially felicitously in a context where the learner is consciously engaged in constructing" (Papert, 1991, p. 1), and not only in just constructing but in constructing something *shareable*. It is a shift in education, from learners as receivers of information, to learners as creators, constructors and main actors; where students engage in activities where they learn *to do mathematics* rather than learn about mathematics (Papert, 1971) — activities where they have *objects-to-think-with* and can develop powerful ideas (Papert, 1980). These are some of the core ideas of constructionism.

An interesting aspect is that, from its inception, constructionism has been related to computer programming, a highly expressive activity. Papert explained, in his seminal work *Mindstorms* in 1980, that in constructing with, or programming the computer, the learner "establishes an intimate contact with some of the deepest ideas from science, from mathematics, and from the art of intellectual model building" (Papert, 1980, p. 5). He also said that "in teaching the computer how to think, [students] embark on an exploration about how they themselves think" (Papert, 1980, p.19). But the emphasis is not only on computer programming, but also on the entire learning culture: A culture or learning environment where there is collaboration, exploration and sharing — again, the sharing idea is very important in constructionism– and where students engage in personally meaningful projects. This "personally meaningful" aspect is central to constructionism.

2.2. Digital technologies for personal expressiveness and the joy of learning

Digital technologies, with their possibility of customization and adaptability are platforms where learners can find and create something personally meaningful, which can bring joy to the learning activity, and thus help them learn best.

Sinclair, Healy and Noss (2015, p. 2) wrote: "A more learnable mathematics should also be one that is worth learning" and using Bruner's expression, they point to the 'sense of delight' (Bruner, 1969, cited in Sinclair et al., 2015) that is involved in many aspects of using digital technologies. But they also warn that

> the technologies offered to students have become increasingly easy to use … and the tasks much more goal-oriented toward the learning of particular concepts. While no one would advocate over-complicated technologies, it is evident that those which offer more expressiveness, though at times they might additionally require a steeper learning curve, also offer more potential — and expressiveness and potential are essential ingredients of both delight and intellectual travel. (Sinclair et al., 2015, p. 2)

There are several important points in the above quote, that I would like to reiterate and comment upon.

First, the observation that the technologies have become increasingly easy to use, while the tasks have become much more goal-oriented. I agree with this, and would add that in doing so, technology-based tasks also lack open-endedness — something which would give learners the opportunity for more creativity and the engagement in more personally meaningful activities.

Their mention of not advocating over-complicated technologies reminded me of another aspect: I argue that, fairly often, something simple, but deep, allows for rich exploration, much more than overly complicated tasks or technologies.

A central point which they then discuss, is that technologies that offer expressiveness have more potential, even if they can be more difficult, and they end with an important idea worth repeating: "expressiveness and potential are essential ingredients of both delight and intellectual travel" (Sinclair et al., 2015, p. 2). In fact, there is a joy in having mathematical insights, but this doesn't mean it is easy. The satisfaction of solving a challenge or problem is much more satisfying than just getting through an activity without facing any challenges.

For example, for decades I was involved in promoting Logo programming in schools; teachers found it difficult, and students also said that it was more difficult than doing things in, for instance, dynamic geometry. Nevertheless, both teachers and students also felt more rewarded when they were able to achieve their goals. In particular, there is the story of a thirteen-year-old boy who told me that he liked Logo because it required thinking, whereas software that just required pointing and clicking, like office suites, made "human beings obsolete" (personal communication, 2009).

However, challenging, creative constructionist implementations are not easy to achieve, particularly in classrooms with established cultures and curricula. As Agalianos et al. (2001 and 2006) pointed out, the use of technologies in schools is shaped by social, economic and political forces and constraints (see also Ruthven, 2008).

One influence on what happens in schools, is the general technology-use culture in today's society. So, let us dwell briefly on what the general technology-use culture and tendencies are, to then analyse what happens in classrooms — that is, what are the prevalent school cultures and practices in terms of technology-integration.

3. Technology-use Cultures in Society and in Schools

3.1. The prevalent culture of technology-use in today's society

Most people today mainly use technology, first, for information and communication: that is, for looking up information (e.g., through Google or Wikipedia); for email and social media (e.g., WhatsApp, Facebook, Instagram); for video conferencing (e.g., Zoom); and for entertainment purposes (e.g., downloading streaming videos from YouTube or Netflix). Those are probably the main uses of technology (and the mentioned apps are among the most popular ones — Most popular apps, 2022).

We can summarize some of the main uses in society as information, communication and entertainment, through *connectivity, social media and mobile media*. Are these three modes present in mathematical educational practices?

In general, even though curricula in many countries around the world emphasize the use of digital technology, there are discrepancies with the reality in classrooms. But before discussing that, let us reflect on the trends, over the last decades, of digital technology use in schools and, in particular, in terms of its use for mathematics.

3.2. School cultures and practices for technology-use in mathematics classrooms in the last decades

In the 1980s, there was, of course, logo — possibly the first educational software, dynamic geometry, computer algebra systems, and spreadsheets — all of which can facilitate, to certain degree, some expressiveness —; although, at the same time, we also had computer assisted instruction, more directed towards practice, tutorials or simulations (Aydin, 2005).

But from the 1990s to the early 21st century, the use of much of the open expressive digital tools and software declined due, for instance, to a lack of adaptability and alignment to conventional classroom practices and curriculum (Ruthven, 2008; see also Agalianos et al., 2001 and 2006); and then, easier-to-use technologies began to dominate (as mentioned by Sinclair et al., 2015). Then, if technology was used in mathematics classrooms, what was prevalent were, either general non-educational and non-mathematical tools (e.g., presentation tools, such as PowerPoint, Word and LaTeX, as I reported in Julie et al., 2009 for the case of Latin-American classrooms), or specific interactive apps for a particular mathematical content or topic. Another use has been the projection of videos (e.g., as reported in Miranda and Sacristán, 2012 and 2013).

Such uses continue until today, although there are new trends towards coding and computational thinking, as discussed further.

We can summarize the observed uses of technology in mathematics classrooms in the past two decades as those:

- *for demonstration/presentation;*
- *for faster computation;*
- *for easier visualization; and*
- *for information*

All of the above are information and communication technologies (ICT). That is not harnessing the full potential of digital technologies, that is not using expressiveness, that is not letting students create and be constructors. It is mostly a teacher-centred use and the tasks are usually the same or similar to paper-and-pencil ones, just with the add-on of technology. In fact, Litke (2020) points to how teacher-centred instructional formats tend to be the norm in the United States, as well as in other nations. Thus, even in developed countries like the U.S.A., teacher-centred instruction is still prevalent.

3.3. Current trends towards coding, computational thinking and different learning approaches

Nevertheless, in the last decade, there are some trends that are more in tune with the precepts of constructionism.

Computational thinking and coding have become a general trend. We have already mentioned at the beginning of the paper how Seymour Papert (1980) considered that computer programming is a way to develop mathematical thinking. The relationship between computational thinking and mathematical thinking is worth dwelling on. One could consider that the computational thinking is part of mathematical thinking, although there are differences. A more profound discussion is beyond of the scope of this paper, although it has been examined by other authors (e.g., Weintrop et al., 2016; Kallia et al., 2021). Carolyn Kieran also provided a very interesting discussion on computational thinking versus mathematical thinking in a plenary panel at the 42nd PME-NA conference (Hoyles et al., 2020), which is worth looking at in full. A main point that she made is that “digital technologies afford multiple varieties of mathematical activity that can offer experiences that involve coding but also those that do not” (Kieran in Hoyles et al., 2020, p. 76). So, there is also a distinction between computational thinking and coding.

Another important trend in the past decade, has been more emphasis on the learning approaches. Previous to that, when promoting computer use, there was little acknowledgement of “the epistemological and cognitive dimensions associated with such change or the complexity associated with the appropriation of tools into mathematical and teaching practices” (Healy, 2006, p. 213) — although Healy was referring to the case of Brazil, this has been the case in many places.

More recently, however, the USA’s 2017 *NMC/CoSN Horizons Report* (Freeman et al., 2017), which aims to identify trends in teaching and learning, pointed to: (i) an increasing use of collaborative learning approaches and of blended learning; shifts from students as consumers to creators and a recent push for coding literacy; (ii) the rise of STEAM learning which seeks to engage students in interdisciplinary learning breaking down traditional barriers between different classes and subjects – this would address one of the criticisms raised by Papert (2006) in his last plenary, as I will discuss further below —; (iii) a rethinking of how schools work, shifting to deeper learning approaches (e.g. project-based learning, etc.) and a redesigning of learning spaces (e.g., ideas like flipped classrooms).

Nevertheless, changes are slow and more so for changing the way mathematics is taught. In 2014, Clark-Wilson and her colleagues pointed out that “[i]nnovative research projects and proposals, and curriculum development don’t seem to have had much impact on students’ learning of mathematics” in the transformative way that was initially anticipated” (Clark-Wilson et al., 2014, p.1). Furthermore, as we will see through the examples given in section 5.1, not only have such innovative projects and development not been able to transform students’ learning and teaching practices, but they are rarely sustained in the long term.

An exception to that is Brock University’s (in Canada) Mathematics Integrated with Computer and Applications (MICA) programme, which has been sustained for over two decades. In this programme, students (mathematics majors and future teachers) learn to design and program interactive and dynamic computer environments (microworlds) to investigate mathematical concepts, conjectures and real-world applications (see Buteau et al., 2016). It is an educational program that has been deemed constructionist; it is also a real world, authentic and sustained implementation.

4. Barriers and Obstacles for Changing Classroom Cultures

However, in general, as we already mentioned, there are social, economic and political forces that are barriers to the uptake and integration of digital technologies and to change of classroom cultures so that these technologies can promote more meaningful teaching and learning. Let us look at that more closely.

4.1. The constraints and inertia of school cultures and systems

As I said at the beginning of the paper, in order to have a meaningful integration of digital technologies in schools, there is a need to change classroom practices and cultures. That is, there is a need to overcome what I call the inertia of current classroom practices, adapt and change them.

In this regard, Seymour Papert, in his plenary talk at the ICMI 17th Study Conference in Vietnam mentioned the followed, which is worth reproducing here:

> Are we going to continue using the new technology to implement what was only there because there wasn't the technology? […] We would never have had airplanes, […] if we had constrained new transportation to follow the schedules of the sailboats and the horse-drawn carriages. That's what we are doing in our schools. […] we have an education system that is rooted in every aspect in the very idea of grades 1, 2, 3, 4, 5, the very idea of cutting up knowledge into the subjects, the order in which to do them, what we do – all this should be put in question. […] Our schools are dictated […] by a technology that's now obsolete, the pencil and paper. Digital technology is the liberator, of allowing completely new things — but, paradoxically, we are caught in a trap of using it to do the same stuff. (Papert, 2006)

What I discussed in section 3.2, and some of main observed uses of technology in mathematics classrooms that I listed there, show that Papert was right: digital technologies are generally used to teach and serve the old; that is, they serve to cater the existing curricula, with much of their potential overlooked.

4.2. Difficulties and determining factors for implementing digital technologies in (mathematics) classrooms

Thus, changing classroom cultures, and overcoming their inertia, is hard. The NMC Horizon Reports consider that the changing role of teachers and educators is difficult, even a "wicked challenge" (Becker et al., 2017). Let us consider what are some of the difficulties, constraints and determining factors that make those changes difficult — in particular for promoting mathematical teaching and learning —, from observations of classroom practices and teachers' experiences. I will begin by drawing some of these from a programme that I was involved with.

Between 1997 to 2007, the Mexican Ministry of Education sponsored what was called the Mexican Teaching Mathematics with Technology (EMAT) programme (see Ursini and Sacristán, 2006; Sacristán and Rojano, 2009), which was carefully designed as a research-based programme; it was also research-linked because much associated studies were carried out. Though the programme was well designed and there were areas of success, we also identified some challenges and difficulties at different levels (see Trigueros and Sacristán, 2008): some at the *teacher, student and classroom level*; some at the *school level*; some at the *local authorities' level*; and some at the *government level* — in fact, it was the federal government that cancelled this programme in 2007 (as described in Trouche et al., 2012; and also, in Sacristán et al., 2021).

So, at the latter level, there can be various policy, administrative and bureaucratic factors that impede successful integration. Some examples are administrators not allowing the use of computers by students; or a lack of support personnel for making sure that computers work or to assist teachers with their use.

At the teacher-student level, some of the issues that have been found (see Sacristán, 2017), are, on the one hand, issues of *time*, where teachers simply lack the time to prepare tasks that integrate technology, or cannot find time within the pre-set curricula to incorporate such tasks.

Moreover, when mathematical digital tools *are* used, sometimes teachers are unable to take advantage of their affordances (maybe due to a deficient Pedagogical Technology Knowledge — PTK —, Thomas and Palmer, 2014). For example, there was a case where we visited a school to see how dynamic geometry was being used (see Sacristán, 2017): The students had to construct bisectors of a triangle and they did it fine, but then they erased it and started all over again. We asked them why. They explained they wanted to make the triangle bigger; that is, they had no idea that they could drag the vertices of the triangle and make the triangle bigger. because the teacher had never conveyed that. So dynamic geometry was not used dynamically; it was just used as a drawing tool. Thus, sometimes the tools' main affordances are not known.

Other observed issues are similar to those identified by Thomas and Palmer (2014), some of which relate to teachers' Pedagogical Technology Knowledge (PTK):

- the teachers' *content knowledge* of mathematics; as well as
- their *attitudes, beliefs and confidence* in and for technology use (also that of their students);
- knowledge of *how to use the technology* both in *technical terms* as well as in *pedagogical* ones (the instrumental genesis of the digital tools for mathematics teaching);
- the *integration of technology tasks with the established curriculum*; and
- *assessment demands* — how do you assess what takes place with technology use?

This is why there is a need for continuous professional development and support in the use of digital technology, as discussed in section 6 below.

All of the above issues, found to impede effective technology integration, are very similar to what a 2011 report listed as the challenges and barriers that UK mathematics teachers' encounter and their concerns for why to use, or not use, technology in their practices:

- a lack of confidence with digital technologies;
- fears about resolving problems with the technology;
- fears about knowing less than their learners;
- access to digital technologies;
- inappropriate training;
- lack of time for preparation;
- a lack of awareness of how technology might support learning;
  not having technology use clearly embedded into schemes of work (Clark-Wilson et al., 2011, p. 20)

That report also includes the following factors among the barriers to a more creative student-focussed use of digital technologies:

- an inadequate guidance concerning the use of technological tools in curriculum documentation;
- assessment practices;
- and "a perception that digital technologies are an add-on to doing and learning mathematics" (Clark-Wilson et al., 2011, p. 6).

A more recent issue is that there is an overload of information and of resources of varying quality. The internet is full of resources and teachers may not know what to choose. In relation to this, Santacruz-Rodríguez identified different types of resource selection criteria by teachers (see Santacruz-Rodríguez and Sacristán, 2019). A first criteria is the technical (also called ergonomic) one, which relates to the ease of use of a resource; others are curricular, mathematical and didactical. Many teachers choose technologies because they fit with something that they want to teach, that is, can be used to cover a specific content. Or they may simply choose a technology because it is easy to use, without necessarily taking much into account mathematical or didactical criteria. In the latter case, the selection criteria could be more for instructional, teacher-centred technology that serves existing (paper-and-pencil) curriculum (which, again, is due to the inertia of classroom cultures).

4.3. School cultures in developed countries vs. those in developing countries

I believe it is also necessary to discuss what happens in developing countries in contrast to developed ones, because additional challenges arise in the first, for technology integration.

In Sacristán et al. (2021), we analysed and compared the situations in India and Mexico. In that chapter, we presented there how there is a lack of equipment and lack of connectivity; and, in rural schools, even a lack of electricity. Computer labs are still common, so students must go to a separate room to use computers; this means that computer technologies are not integrated with and within disciplinary topics, such as mathematics. There is also a lack of professional development for teachers.

For example, in a survey partly reported in that chapter (Sacristán et al, 2021), in rural schools in the region of Oaxaca, Mexico, half of the schools did not have access to internet, and some schools had only two computers for the entire school.

Thus, in most developing countries, there are issues of restricted access. Prevalence of old hardware seems to be the norm, so teachers have to cope with what they have. Or, when there is only a computer lab, they often cannot use it for mathematics classes, because it is used exclusively for computer science.

We have even documented the case where computer labs were used as storage rooms (Herrera-Salgado, 2011). Or there are extreme cases, such as one in Ghana, where computer science is taught using chalk blackboards because the minimum hardware is not available (see Sacristán et al., 2018).

Thus, there are issues that affect digital integration such as: digital gaps and fears; access (particularly in rural schools); and issues of professional development. These examples show the discrepancies and the digital gap between developed and developing countries. Thus, that which is readily available and taken for granted in some countries, is scarce in others.

At the same time, however, there are a few commonalities in all countries. In particular, it seems that the transition to meaningful integration of digital technologies for mathematical teaching and learning has been much slower than anticipated.

4.4. Political and social forces (e.g., the Covid-19 pandemic)

Another important discussion point is that of policy. Top-down policies can generate some changes, but so do changes in society. For instance, the changes that have happened due to the Covid-19 pandemic have pushed the use of technology, although perhaps not in the most significant or favourable ways.

In Mexico, in the late 1990s, there was a push to insert digital technologies in schools by the government, with the EMAT (mentioned above) and other parallel or related programmes (e.g., Enciclomedia — see Trouche et al., 2012, and Sacristán et al, 2021). Then, in 2007, the government stopped those important programmes, and other smaller initiatives were not very successful.

Thus, when the Covid-19 pandemic hit, schools were unprepared to deal with the necessary distance education. Elementary and middle-schools relied on television programs and YouTube videos. And teachers used WhatsApp to send homework to students and vice versa (students sent homework back to the teachers). That is, technology was not being used to change the way mathematics is taught, nor did it encourage mathematical learning. On the other hand, it is a social situation that pushed for more technology use, in particular, for online teaching and learning. It would be interesting if we could harness that momentum to veer educational practices and cultures towards more innovative uses of digital technologies in schools.

5. Harnessing the Potentials of Digital Technologies for Creating Innovative School Cultures

Something that was called for by Seymour Papert (2006) in his keynote at the ICMI 17th Study Conference in Vietnam, was that we should devote 10% of our time to reflect on how technology can create new mathematical ideas and practices –something which we have dubbed Papert's 10%.

Thus, one aspect that we need to consider is how take advantage of prevalent technologies and trends, if we are to break away from what he criticized, when he said that what we do in schools is like operating airplanes with schedules of horse-drawn carriages (Papert, 2006). How can we take advantage, for instance, of the trend towards coding and computational thinking? Or, how can we take advantage of connectivity, mobile technologies and social networks (mentioned in section 3.1) for mathematical activities, rather than just for communicating (such as the non-mathematical use during the pandemic in Mexico)?

In the following section, I present some selected examples of constructionist implementations that have taken advantage of connectivity.

5.1. Selected examples of "connected" constructionist projects

The first example is not exactly a project or implementation, but an example worth discussing: that of the Scratch programming environment (https://scratch.mit.edu/). Scratch is a beautiful community where people share their projects online, programmed in Scratch; children are creating in Scratch and they are remixing them. In https://scratch.mit.edu/statistics/one can see the trends of how Scratch has been used; its use has been increasing over the years and during the Covid-19 pandemic, it's increase was huge. Yet, if we ask how mathematical is its use? The answer is probably not that much. In the UK, however, the ScratchMaths curricula (http://www.ucl.ac.uk/scratchmaths) was developed in an attempt to bridge computer programming in Scratch with mathematical thinking and learning. The emphasis of ScratchMaths may have been more on computer programming for mathematics, rather than on the social collaboration through the connectivity of Scratch, but it is still worth mentioning. Some results of ScratchMaths are reported in Benton et al. (2017). One interesting point that arose from ScratchMaths, is the issue of *fidelity* (see Hoyles et al., 2020): No matter how well designed a programme is, there is a question of how it is then implemented by the teachers; that is, there is a gap between the intent of the developers and the understanding the teacher.

Thus, technologies are often appropriated in ways unanticipated by their developers and may not yield the expected results (they may even go against some of their fundamental principles; for instance, when in constructionist designs, students are not allowed to create).

Other examples of "connected" constructionist projects are older, but they are worth presenting. One is the Weblabs European project (WebLabs, 2011; Noss and Hoyles, 2005) that took place from 2002 and 2005, and in which I was fortunate to participate. Students, 10 to 14 years old, from several countries across Europe, engaged in scientific and mathematical investigations. The project investigated new ways of representing expressive mathematical and scientific knowledge. Students programmed models of their ideas using the non-text-based computer programming environment ToonTalk (http://www.toontalk.com). One very interesting thing was that the participants collaborated online, sharing and discussing their investigations and constructs using a type of blogs called WebReports, which was something that was very innovative at the time. Unfortunately, it didn't continue after the project ended.

Inspired by WebLabs, we developed a project in Mexico, which we called the iMat online virtual mathematics laboratory, which was a distance education learning environment (see Olivera and Sacristán, 2012). In this project, university students engaged in collaborative mathematical explorations through model-eliciting (Lesh et al., 2000) activities, to discuss and reflect upon various types of real-life mathematical problems. One of the difficulties that our team encountered was in designing the activities. It was very difficult to break the inertia of how established curricular tasks are commonly structured and come up with innovative designs that really engaged learners. After months of struggle, we came up with activities related to selected themes (linear motion; free-fall and gravity; population growth; cryptography), where students were involved in tasks such as analysing videos and other data, and constructing models (e.g., of the gravity on the Moon). Digital technologies (e.g., video software for frame-by-frame analysis; a virtual ruler for measurements; spreadsheets or CurveExpert (http://www.curveexpert.net) for finding mathematical equations to fit the data; and modelling software, such as Modellus — https://modellusfq.blogspot.com/) were used in such tasks; as well as for collaborating, sharing, discussing online (as was done in WebLabs) and proposing new explorations, through a web-based discussion forum. The experiences were very rich and rewarding, but after a couple of years iMat was discontinued because it required too much effort and did not fit with the established curriculum.

This is what tends to happen. There are many wonderful innovative projects and programmes across the world, but there is a lack of continuity. More often than not, the projects' funding ends, they are abandoned, and there is rarely any uptake by others. For instance, the WebLabs files can now only be found in archive.org's Wayback Machine (WebLabs, 2011).

In fact, many interesting projects remain unknown and do not get much projection beyond some publications. Related to that latter point, I must also mention the very interesting work carried out for many years by Chronis Kynigos and his colleagues at the Educational Technology Lab of the University of Athens, Greece (http://en.etl.eds.uoa.gr/educational-technology-lab-etl.html). They have developed wonderful constructionist materials (software and projects) over several decades, but they lack resources — as does MIT to disseminate Scratch — so their work has not expanded much beyond their group.

So, even when there are wonderfully designed innovations, we see other social forces and trends (lack of continuity, of support, or issues of fidelity) that restrict their possible successful implementations in schools.

5.2. Can digital technologies serve as catalysts to change school cultures?

Hoyles pointed out that her interest in digital technologies has been to “help learners open windows to mathematical knowledge by using digital technologies in innovative, future-oriented and intellectually rigorous ways” (in Hoyles et al., 2020, p. 70). I believe that digital technologies can act and help to create new cultures in the classroom. But the questions remain: How can we promote the necessary changes? How do we break with the inertia of school cultures to harness the potentials of digital technologies for mathematical thinking and learning and so they can serve as catalysts for creating new school cultures?

One aspect is that it is important to start with what is already being done. That is, to work with and within the school systems and curricula. Curricula and classroom cultures are not going to going to change soon, so we need to work within them, rather than against them. But there has to be an openness to change.

If there is some openness, one approach to change, is to gradually adapt new pedagogies and designs to integrate digital technologies in innovative (e.g., constructionist) ways. For that, I particularly like structured environments that follow the extended microworlds idea proposed by Hoyles and Noss (1987), where a microworld takes into account several components: the student, the context in which the learning takes place, the pedagogy, which includes the teacher and how they orchestrate all the didactical materials, as well as the technology itself, that is the technical aspect (see also Sacristán et al, 2009).

But none of this can be achieved without support from the authorities and, most importantly, without considering the teachers (promoting their involvement in generating change, resources and decision-making), as well as supporting and training them (professional development). Thus, I continue by focusing on the role and importance of the teachers for generating technology integration in math classrooms.

6. The Role and Importance of the Teacher

The teacher is the key player for successful implementations of technology-centred educational innovations. However, as Paul Goldenberg said, it is necessary to “provide instruction and time for teachers to become creative users of the technology” (Goldenberg, 2000, p.8). We saw that, in the EMAT programme, where even the most motivated and supported teachers — who were directly and continuously supported by us — that it took them three years to actually grasp and change their ways of teaching, and to change their classroom cultures (Trigueros and Sacristán, 2008).

Thus, changes need to take place in gradual steps, and fit in with what is already being done. We have also discussed, in section 5.1, the issue of fidelity (see Hoyles et al., 2020), where teachers need to develop an understanding of innovative designs in order to implement them with more fidelity to the main principles, in order to ensure a higher probability of success. Wright (in Aldon et al., 2017, 54:02) gave the following points to be taken into account to help teachers adopt innovative technologies: *ease of use; small steps* — that fit well into teachers' existing practice —; have a *perceived immediate gain* to students' learning; and support (technical support, support from a professional learning community, and *support* from someone who will give initial boost to the innovation and sustain its promotion).

This is why continuous professional development (PD) and support in the use of digital technology is of upmost importance. As Celia Hoyles mentioned at the ICTMT 2017 in Lyon, France: "You will never achieve things in the classroom without proper professional development" (C. Hoyles, personal communication, July 2017). As I will discuss further below, that is also related to the need to build communities of teachers. In Faggiano et al. (2021), we identified some theories that inform the design of professional development programs for integrating digital technologies in mathematics education, and which provide insights for future PD implementations. We should also consider the role of connectivity and distance education, and changing the role of the teachers and their professional development

Thus, we need to provide teachers with professional development but, at the same time, also involve them as active collaborators in generating the changes for meaningful technology integration; that is, involve them in designing resources and taking decisions.

When teachers are involved as co-creators (working *with* teachers), they can appropriate themselves better of new resources: if they feel they are participants themselves, they have more ownership, and the appropriation is easier to achieve. For that, there needs to be collaboration with researchers, so that teachers feel they are also decision-makers; this is crucial in terms of motivation, in affecting their beliefs and overcoming their fears and apprehension. Also, involving teachers in the design process, may help them improve their Pedagogical Technology Knowledge (PTK).

Pepin et al. (2017) explain that teachers' design capacity of resources depends and can be refined by (i) how they understand and transform existing resources ("re-mixing") to (re-) design instruction; (ii) the organization of collectives of teachers and designers, not only for design, but to share and observe other teachers' experiences; and (iii) multi-national efforts to create quality digital resources.

The communities, collectives or networks, in which teachers and designers can participate, can also be online ones (e.g., France's Sésamath — http://www.sesamath.net/ — see Trouche et al, 2012).

In terms of the multi-national efforts to create quality digital resources, a notable mention is the European Mathematical Creativity Squared (MC2) Project (http://www.mc2-project.eu), which had amongst its aims, to rethink the nature of open educational resources, create Communities of Interest in four European countries, and collectively design and produce digital content for creative mathematical thinking (M C Squared Project, n.d.).

Thus, teachers' collectives or communities of practice, where they can share, reflect and be supported by peers, experts and researchers, are important. I personally have been involved in a couple of experimental programmes where we have had in-service teachers reflecting and collaborating in small communities (e.g., Parada et al., 2013; another is mentioned in Sacristán et al., 2011 and Trouche et al., 2012). Participation in communities where teachers can share and reflect on their practice, seems to help and enrich the integration of technologies in their practices and create the needed changes in the classroom cultures.

To work with teachers is also a way to involve them in Papert's 10%, that is, for them to reflect on what new knowledge can emerge from the use of technology (Papert, 2006); to think about their own learning and their students learning; and to think differently, so that they can innovate and change school cultures.

7. Concluding Remarks

To conclude, I would like to summarize some of the main points discussed in this paper:

- The inertia of the classroom and the paper-and-pencil cultures limit change.
- The teacher is a key player for successful and transformative technology integration.
- However, we need to promote models of collaboration (such as communities and networks) between teachers, researchers and policy-makers: (i) to enhance teachers' professional development, (ii) to empower them, and (iii) to provide means for sharing, discussing and improving resources and their implementation.
- The educational systems also need to change and provide flexibility for teachers to have time to engage in collaboration and innovation.

It is thus that we may be able to "ride the wave" with society's trends in technology-use to harness them and veer them towards meaningful mathematical learning opportunities and new practices. In particular, mobile technologies are more accessible in many regions and social strata, so we need ways to take advantage of them for more mathematical practices, teaching and learning. In general, we should find ways to shift the emphasis in classrooms from technologies for communication, information and presentation, to technologies that promote mathematical thinking.

I end with a call to action, akin to Papert's 10%, to reflect on how else we can use digital technologies to be catalysts for innovation of mathematical practices and learning, and that change school cultures.

References

A. Agalianos, R. Noss, and G. Whitty (2001). Logo in mainstream schools: the struggle over the soul of an educational innovation. *British Journal of Sociology of Education*, 22(4), 479–500.

A. Agalianos, G. Whitty, and R. Noss (2006). The social shaping of Logo. *Social Studies of Science*, 36, 241–267.

G. Aldon, B. Barzel, M. Panero, C. Sabena, and D. Wright (2017, July 5). Formative assessment and technology (round table) [Video]. *13th International Conference on Technology in Mathematics Teaching (ICTMT 13)*, ENS-Lyon, France. https://youtu.be/P04e3IN38xA Also accessible via https://ictmt13.sciencesconf.org/resource/page/id/16.html

D. E. Aydin (2005). The use of computers in mathematics education: a paradigm shift from "computer assisted instruction" towards "student programming". The Turkish Online *Journal of Educational Technology*, 4(2), 27–34. http://www.tojet.net/articles/v4i2/424.pdf

S. A. Becker, M. Cummins, A. Davis, A. Freeman, C. Glesinger Hall, C., and V. Ananthanarayanan (2017). *NMC Horizon Report: 2017 Higher Education Edition*. Austin, Texas: The New Media Consortium. https://www.learntechlib.org/p/174879/

L. Benton, C. Hoyles, I. Kalas, and R. Noss (2017). Bridging Primary Programming and Mathematics: Some Findings of Design Research in England. *Digital Experiences in Mathematics Education*, 3, 115–138. https://doi.org/10.1007/s40751-017-0028-x

C. Buteau, E. Muller, N. Marshall, A. I. Sacristán, and J. Mgombelo (2016). Undergraduate Mathematics Students Appropriating Programming as a Tool for Modelling, Simulation, and Visualization: A Case Study. *Digital Experiences in Mathematics Education*, 2, 142–166.

A. Clark-Wilson, A. Oldknow, and R. Sutherland (Eds.) (2011). Digital technologies and mathematics education: A report from a working group of the Joint Mathematical Council of the United Kingdom. https://www.jmc.org.uk/documents/JMC_Report_Digital_Technologies_2011.pdf

A. Clark-Wilson, O. Robutti, and N. Sinclair (Eds.) (2014). *The Mathematics Teacher in the Digital Era: An International Perspective on Technology Focused Professional Developmen*t. Springer. https://doi.org/10.1007/978-94-007-4638-1_1

E. Faggiano, H. Rocha, A. I. Sacristán, and M. Santacruz-Rodríguez (2021). Towards pragmatic theories that underpin the design of teacher professional development concerning technology use in school mathematics. In A. Clark-Wilson, A. Donevska-Todorova, E. Faggiano, J. Trgalová, and H.-G. Weigand (Eds.), *Mathematics Education in the Digital Age: Learning, Practice and Theory*. Routledge, pp. 42–68. https://doi.org/10.4324/9781003137580-4

A. Freeman, S. Adams Becker, and M. Cummins (2017). *NMC/CoSN Horizon Report: 2017 K-12 Edition*. Austin, Texas: The New Media Consortium. Retrieved April 14, 2022 from https://www.learntechlib.org/p/182003/

P. Goldenberg (2000). Thinking about technology in Math Classrooms; Educational Development Centre. *Issues in Mathematics Education*. Education Development Center, Inc. http://ltd.edc.org/sites/ltd.edc.org/files/ThinkTalkTech.pdf

L. Healy (2006). A developing agenda for research into digital technologies and mathematics education: A view from Brazil. In C. Hoyles, J.-B. Lagrange, L. H. Son y N. Sinclair (Eds.), *Proceedings of the Seventeenth ICMI Study Conference "Technology Revisited" — Part 2: Contributions*. Hanoi Institute of Technology & DIDIREM-Université Paris 7, pp. 213–220. https://www.mathunion.org/fileadmin/ICMI/files/Digital_Library/icmi-study17/ ICMI17proceedingsPart2.pdf

M. A. Herrera-Salgado (2011). Cambios en el aula con el uso de tecnología, y resolución de problemas algebraicos [Masters thesis]. Cinvestav-IPN, Mexico. http://www.matedu.cinvestav.mx/~asacristan/Tesistas/MiguelHerreraTesis.pdf

C. Hoyles and R. Noss (1987) Synthesising Mathematical Conceptions and their Formalisation through the Construction of a Logo-based School Mathematics

Curriculum. International *Journal of Mathematics Education in Science and Technology*, 18, 4, 581–595.

C. Hoyles, C. Kieran, T. Rojano, A. I. Sacristán, and M. Trigueros (2020). Reflections on digital technologies in mathematics education across cultures. *Mathematics Education Across Cultures: Proceedings of the 42nd Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education*, 69–92. https://doi.org/10.51272/pmena.42.2020-1

C. Julie, A. Leung, N. C. Thanh, L. S. Posadas, A. I. Sacristán, and A. Semenov(2009) Some Regional Developments in Access and Implementation of Digital Technologies and ICT. In: Hoyles C., Lagrange JB. (eds) Mathematics Education and Technology-Rethinking the Terrain. The 17th ICMI study. *New ICMI Study Series*, vol 13. Boston, MA: Springer, pp. 361–383.https://doi.org/10.1007/978-1-4419-0146-0_17

M. Kallia, S. P. van Borkulo, P. Drijvers, E. Barendsen, and J. Tolboom (2021). Characterising computational thinking in mathematics education: A literature-informed Delphi study. *Research in Mathematics Education*, 23(2), 159–187. https://doi.org/10.1080/14794802.2020.1852104

R. Lesh, M. Hoover, B. Hole, A. Kelly, T. Post (2000). Principles for Developing Thought-Revealing Activities for Students and Teachers. In A. Kelly, R. Lesh (Eds.), *Research Design in Mathematics and Science Education* (pp. 591–646). Lawrence Erlbaum Associates.

E. Litke (2020). Instructional practice in algebra: Building from existing practices to inform an incremental improvement approach. *Teaching and Teacher Education*, 91, 103030. https://doi.org/10.1016/j.tate.2020.103030

M C Squared project — Project (n.d.). http://www.mc2-project.eu/index.php/the-project

M. L. Miranda and A. I. Sacristán (2012). Digital technologies in Mexican high-schools. In L. R. Van Zoest, J.-J. Lo, and J. L. Kratky (Eds.). *Proceedings of the 34th annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education*. Kalamazoo, MI: Western Michigan University, pp. 1097–1102.

M. L. Miranda and A. I. Sacristánb (2013). Lack of sense of purpose in the use or technology for mathematical teaching. In E. Faggiano and A. Montone (Eds.), *Proceedings of the 11th International Conference on Technology in Mathematics Teaching (ICTMT11)*. Bari, Italia: University of Bari (Università degli Studi di Bari Aldo Moro), pp. 200–205.

Most popular apps (2022). (2022, January 11). *Business of Apps*. https://www.businessofapps.com/data/most-popular-apps/

R. Noss and C. Hoyles (2005). Exploring mathematics through construction and collaboration. In R. Sawyer (Ed.), *The Cambridge Handbook of the Learning Sciences (Cambridge Handbooks in Psychology)*. Cambridge University Press, pp. 389–406. https://doi.org/10.1017/CBO9780511816833.024

R. Noss and C. Hoyles (2019). Microworlds, constructionism and mathematics. *Educación Matemática*, 31(2). http://www.revista-educacion-matematica.org.mx/descargas/vol31/ 2/01_REM31-2_i.pdf

M. Olivera and A. I. Sacristán (2012) An Online Setting for Exploring, Constructing, Sharing and Learning Mathematical Ideas. In C. Kynigos, J. Clayson and N. Yiannoutsou (Eds). *Constructionism: Theory, Practice and Impact. Constructionism 2012 Conference Proceedings*. Athens, Greece: ETL, National and Kapodistrian University of Athens, pp. 385–394.

S. Papert (1971). Teaching Children to be Mathematicians vs. Teaching about Mathematics. *Artificial Intelligence Memo Number 249*. http://hdl.handle.net/1721.1/5837

S. Papert (1980). *Mindstorms: Children, Computers and Powerful Ideas*. Basic Books.

S. Papert (1991). Situating constructionism. In I. Harel and S. Papert (Eds.), *Constructionism*. Ablex Publishing Corporation, pp. 1–8. http://www.papert.org/articles/Situating Constructionism.html

S. Papert (2006, December 4). From the math wars to the new new math [Keynote Lecture]. 17th ICMI Study Conference: Mathematics Education and Digital Technologies, Rethinking the Terrain. Hanoi, Vietnam. Transcript available at http://dailypapert.com/wp-content/uploads/2012/05/Seymour-Vietnam-Talk-2006.pdf, https://drive.google.com/file/d/0BzmXPBo2CiPTNjJMOGlDc2dvVDA

S. Parada, F. Pluvinage, and A. I. Sacristán (2013). Reflexiones en una comunidad práctica de educadores matemáticos sobre los números negativos. *Recherches en Didactique des Mathématiques*, 33(3). 233–267.

B. Pepin, G. Gueudet, and L. Trouche (2017). Refining teacher design capacity: Mathematics teachers' interactions with digital curriculum resources. *ZDM*. https://doi.org/10.1007/s11858-017-0870-8

K. Ruthven (2008). Mathematical technologies as a vehicle for intuition and experiment: A foundational theme of the International Commission on Mathematical Instruction, and a continuing preoccupation. *International Journal for the History of Mathematics Education*, 3(2), 91–102

A. I. Sacristán (2017). Digital technologies in mathematics classrooms: Barriers, lessons and focus on teachers. In E. Galindo and J. Newton (Eds.), *Proceedings of the 39th annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education*. Hoosier Association of Mathematics Teacher Educators, pp. 90–99. http://www.pmena.org/pmenaproceedings/PMENA%2039%202017%20Proceedings.pdf

A. I. Sacristán, N. Calder, T. Rojano, M. Santos-Trigo, A. Friedlander, H. Meissner, and M. Tabach (2009). The Influence and Shaping of Digital Technologies on the Learning — and Learning Trajectories — of Mathematical Concepts. In C. Hoyles and J.-B. Lagrange (Eds.), *Mathematics Education and Technology — Rethinking the Terrain*. Springer, pp. 179–226. https://doi.org/10.1007/978-1-4419-0146-0_9

A. I. Sacristán, L. Kaminskienė, M. Sabin, and R. A. K. Baafi (2018). WG7: Constructionism in Upper Secondary and Tertiary Levels. Constructionism 2018 (August 20–25, 2018): Constructionism, Computational Thinking and Educational Innovation: Conference Proceedings. Vilnius University (Faculty of Philosophy and Institute of Data Science and Digital Technologies) / Lithuanian Computer Society, pp. 925–939. http://www.constructionism2018.fsf.vu.lt/ proceedings

A. I. Sacristán, I. Sandoval, and N. Gil (2011). Teachers engage in peer tutoring and course design inspired by a professional training model for incorporating technologies for mathematics teaching in Mexican schools. In M., Joubert; A., Clark-Wilson, and M., McCabe (Eds.) *Enhancing Mathematics Education Through Technology. Proceedings of the Tenth International Conference on Technology in Mathematics Teaching*. Portsmouth, UK: University of Portsmouth, pp. 249–254.

A. I. Sacristán and T. Rojano (2009). The Mexican National Programs on Teaching Mathematics and Science with Technology: The Legacy of a Decade of Experiences of Transformation of School Practices and Interactions. In A. Tatnall and A. Jones (Eds.), *Education and Technology for a Better World.* IFIP Advances in Information and Communication Technology, Vol. 302. Springer, pp. 207–215.

A. I. Sacristán, J. Rahaman, S. Srinivas, and T. Rojano (2021). Technology integration for mathematics education in developing countries, with a focus on India and Mexico. In A. Clark-Wilson, A. Donevska-Todorova, E. Faggiano, J. Trgalova, and H.-G. Weigand (Eds.), *Mathematics Education in the Digital Age: Learning, Practice and Theory*. Routledge, pp. 186–212. https://doi.org/10.4324/ 9781003137580-12

M. Santacruz-Rodríguez and A. I. Sacristán (2019). Criterios de selección de recursos digitales para la enseñanza de la geometría en primaria/Criteria of digital resource selection for the teaching of geometry in primary school. In Otten, S, Candela, A. G., de Araujo, Z., Haines, C., and Munter, C. (Eds.), *Proceedings of the 41st annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education*. University of Missouri, pp. 1678–1686. https://www.pmena.org/pmenaproceedings/PMENA%2041%202019%20Proceedings.pdf

N. Sinclair, L. Healy, and R. Noss (2015) Editorial. *Digital Experiences in Mathematics Education 1*, (1), 1–6. https://doi.org/10.1007/s40751-015-0006-0

M. O. J. Thomas and J. Palmer (2014). Teaching with digital technology: Obstacles and opportunities. In A. Clark-Wilson, O. Robutti, and N. Sinclair (Eds.), *The Mathematics Teacher in the Digital Era: An International Perspective on Technology Focused Professional Development*. Mathematics Education in the Digital Era, Vol 2. Springer, pp. 71–89. https://doi.org/10.1007/978-94-007-4638-1_4

M. Trigueros and A. I. Sacristán (2008). Teachers' practice and students' learning in the Mexican programme for Teaching Mathematics with Technology. International *Journal of Continuing Engineering Education and Life-Long Learning (IJCEELL)* Vol. 18 (5/6): 678–697. https://doi.org/10.1504/IJCEELL.2008.022174

L. Trouche, P. Drijvers, G. Gueudet, and A. I. Sacristán (2012). Technology-Driven Developments and Policy Implications for Mathematics Education. In M. A. (Ken) Clements, A. J. Bishop, C. Keitel, J. Kilpatrick, and F. K. S. Leung (Eds.), *Third International Handbook of Mathematics Education (Chap. 24)*. Springer International Handbooks of Education (Vol. 27). Springer, pp. 753–789. https://doi.org/10.1007/978-1-4614-4684-2_24

S. Ursini and A. I. Sacristán (2006): On the role and aim of digital technologies for mathematical learning: experiences and reflections derived from the implementation of computational technologies in Mexican mathematics classrooms. In C. Hoyles, J.-B. Lagrange, L. H. Son, and y N. Sinclair (Eds.), *Proceedings of the 17th ICMI Study Conference "Technology Revisited" — Part 2: Contributions* (pp. 477–486). Hanoi Institute of Technology & DIDIREM — Université Paris 7. https://www.mathunion.org/fileadmin/ICMI/files/Digital_Library/icmi-study-17/ICMI17proceedingsPart2.pdf

WebLabs: New representational infrastructures for e-learning. (2011, August 31). https://web.archive.org/web/20110831034948/http://www.lkl.ac.uk/kscope/weblabs/index.htm

D. Weintrop, E. Beheshti, M. Horn, K. Orton, K. Jona, L. Trouille, and U. Wilensky (2016). Defining computational thinking for mathematics and science classrooms. *Journal of Science Education and Technology*, 25(1), 127–147. https://doi.org/ 10.1007/s10956-015-9581-5